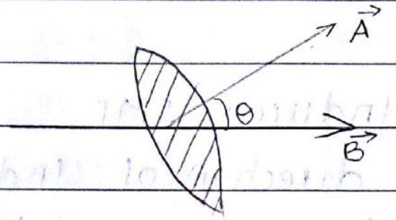
Electromagnetic Induction.

Magnetic flux - The total no. of magnetic field lines passing normal to a surface is called the magnetic flux. It can be measured as the dot product of intensity of magnetic field ($\vec{B}$) and area ($\vec{A}$). i.e $\phi = \vec{B}.\vec{A}$

$= BA\cos\theta$



Magnetic flux is a scalar quantity and its unit is weber

$1\ Wb = 1\ Tm^2$

Electromagnetic induction - whenever the magnetic flux linked with a closed circuit changes an EMF is induced in the circuit this phenomenon is called EMI.

EMF can be induced in the following 3 different ways

- By the relative motion b/w the coil and the magnet called the dynamo effect.
- EMF can be induced in a coil by changing the current through a neighbouring coil called the transformer effect
- By changing the relative orientation of the coil with respect to the magnet.

Faraday's laws of EMI

First law - whenever a magnetic flux linked with a closed circuit changes an EMF is induced in the circuit. The induced EMF lasts as long as the change in the flux lasts.

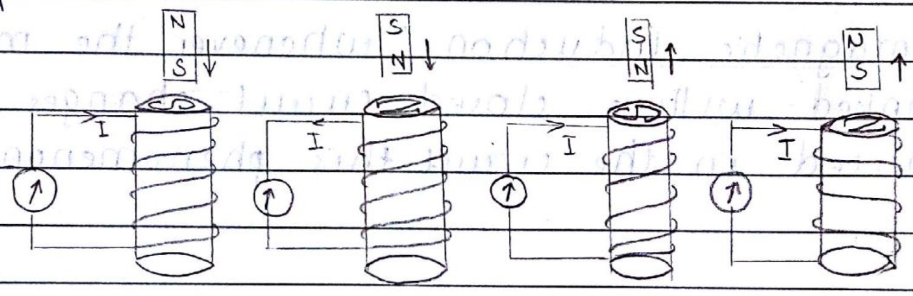
Second law - The magnitude of induced EMF is directly propotional to the rate of change of magnetic flux linked with the circuit.

$$E \propto \frac{d\phi}{dt} \quad (\phi)$$

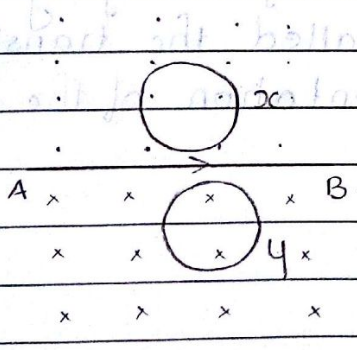
$$E = -\frac{d\phi}{dt}$$

LENZ's law - Direction of Induced EMF

lenz's law states that the direction of Induced EMF is such that it opposes the change which produces it



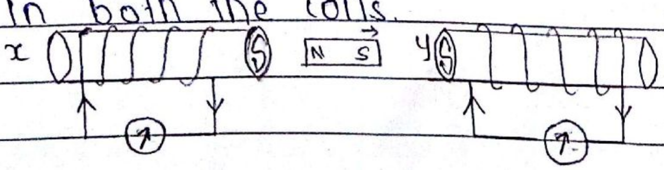
Q - Determine the direction of Induced current in the circular loops x and Y when current from A to B Increases, (ii) decreases (iii) remains the same.



Direction of field
outward - x
Inward - y
lenz's law

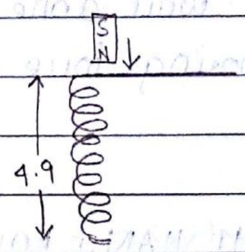
x	y
(i) clockwise.	(i) anticlockwise
(ii) anticlockwise	(ii) clockwise
(iii) No Induced EMF	(iii) No Induced EM

Q A bar magnet is moving b/w 2 coils x and y as shown in the figure. find the direction of Induced current in both the coils.



Q A bar magnet is allowed to fall through a solenoid of vertical height 4.9 m. find the time taken by the magnet to cross the solenoid.

(a) 0.7 s
(b) 0.8 s
(c) 0.9 s
(d) 1 s
(e) 1.1 s



ans $s = ut + \frac{1}{2}at^2$

$4.9 = \frac{1}{2} \times \cancel{9.8}^{4.9} t^2$

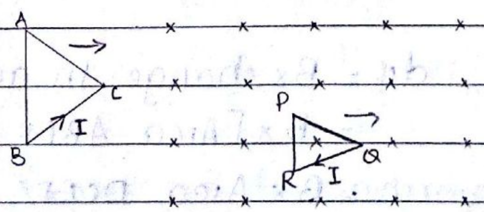
$t = 1$ Sec

but ans: 1.1 s (because of the opposing force, $a < 9.8$)

Q A barmagnet and a wooden block are allowed to fall from the same height through a metal ring. which one will reach the bottom first and why?

ans wooden block will reach the bottom first because magnetic block will experience an opposing force.

Q 2 conducting loops ABC and PQR are moving in a region of magnetic field as shown in the figure. find the direction of the induced current in the loops.



ABC in anticlockwise direction
PQR in clockwise direction.

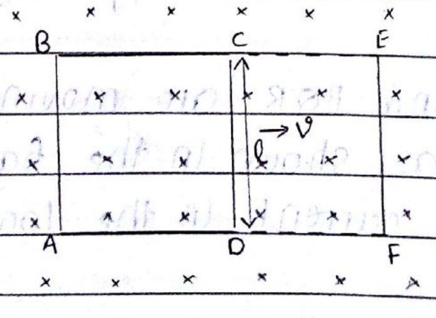
Q Lenz's law is an example for the law of conservation of energy. why?

ans the mechanical work done in moving the magnet against the opposing force is converted into electrical energy.

FLEMINGS RIGHT HAND RULE - stretch the first 3 fingers of the right hand in mutually ⊥' directions. If the forefinger indicate the direction of magnetic field and the thumb indicate the direction of motion of the conductor then the middle finger will give the direction of induced current.

Motional EMF

Figure shows a rectangular conducting loop ABCD, kept in a uniform magnetic field of intensity B with the plane of the loop $\perp^r$ to the magnetic field. The side CD of length l, is moving towards right with a velocity v and takes a new position EF in a time dt.



we have the magnetic flux

$\phi = \vec{B}\cdot\vec{A}$

$= BA\cos\theta$

$= BA \quad (\because \theta = 0)$

$\therefore$ change in flux, $d\phi = B \times$ change in area

$= B \times [\text{Area } ABEF - \text{Area } ABCD]$

$= B \times \text{Area } DCEF$

$= B \times CD \times DF$

$= B \times l \times v\,dt$ [Displacement = velocity × time] we have Induced EMF

$$E = \frac{-d\phi}{dt}$$

$$= \frac{-Bl v dt}{dt}$$

$$\boxed{E = -Blv}$$

The EMF Induced In a conductor due to its motion In a magnetic field is called Motional EMF

NOTE
If the velocity of the conductor is making an angle θ with the magnetic field, $E = -Blv\sin\theta$

Induced Current

let R be the resistance of the conductor, then Induced current $I = -\frac{Blv}{R}$

we have Force $F = IlB$

$$= -\frac{Blv \times lB}{R}$$

$$\boxed{F = -\frac{B^2 l^2 v}{R}}$$

Power $P = F \times V$

$$= -\frac{B^2 l^2 v \times v}{R}$$

$$P = -\frac{B^2 l^2 v^2}{R}$$

Expression for Induced charge

$$E = \frac{-d\phi}{dt}$$

$$I.R = -\frac{d\phi}{dt}$$

$$\frac{dQ}{dt} \times R = -\frac{d\phi}{dt}$$

$$dQ = -\frac{d\phi}{R}$$

Induced charge = $\frac{\text{change in flux}}{\text{Resistance}}$

Q A train is moving on a railway track of width 1.2 m with a speed of 72 km/hr find the EMF Induced across the axle of the train (Earth's magnetic field = 0.4×10^{-4} Tesla)?

$$E = -Bl v$$

length = width of the railway track

$$= -0.4 \times 10^{-4} \times 1.2 \times 72 \times \frac{5}{18}$$

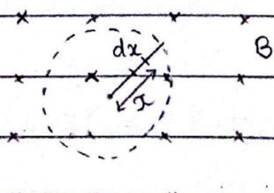
$$= -0.4 \times 20 \times 1.2 \times 10^{-4}$$

$$= -4 \times 2.4 \times 10^{-4}$$

$$= -9.6 \times 10^{-4} \text{ V}$$

Impo.

* Q A conductor of length l is rotating in a uniform magnetic field of intensity B. above one of its ends with a uniform angular velocity ω in such a way that the length is ⊥ to the magnetic field. find the EMF Induced across the ends of the conductor



$$V = x\omega$$

$$dE = -B\, dx . x\omega$$

$$= -B\omega x\, dx$$

$$E = \int_0^l -B\omega x\, dx$$

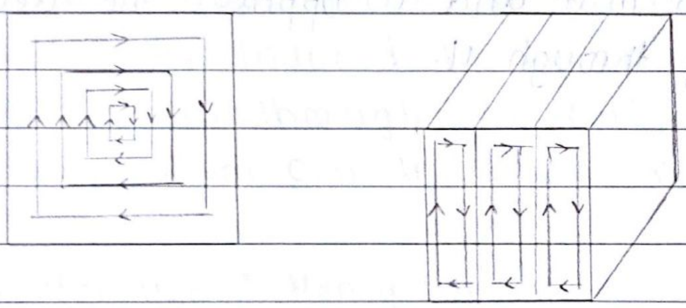
$$= -B\omega \int_0^l x\, dx$$

$$E = -B\omega\left[\frac{x^2}{2}\right]_0^l$$

$$= -B\omega\left[\frac{l^2}{2} - 0\right]$$

$$\boxed{E = -\frac{1}{2}B\omega l^2}$$

Eddy Currents - whenever the magnetic flux linked with a sheet of metal or block of metal changes, an EMF is induced in it. The current flows in closed paths inside the metal such currents are called eddy currents.



Eddy currents heats the metal to reduce this heating effect, the block is made into thin sheets called lamination and are seperated by some insulated materials like paper, varnish, etc

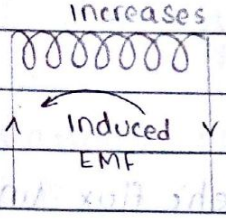
Applications of eddy currents

1. In Induction furnaces.
2. Electric brakes
3. Speedo meter
4. For producing electromagnetic damping

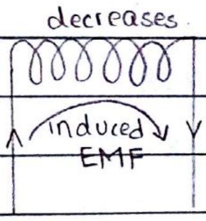
Self Induction - whenever the magnetic flux linked with a coil changes due to the ~~cur~~ change in current flowing through it, an EMF is induced in the same coil. This phenomenon is called self induction.

The EMF induced is known as back EMF.

when the current flowing through a ~~current~~ coil increases, the flux linked with it also increases. As a result an EMF is induced and it opposes the growth of current flowing through it



When Current flowing through a coil decreases, the flux linked with it also decreases. As a result an EMF is induced in it and it opposes the decay of current flowing through it.



∴ Self Induction can be defined as the prop. of the coil by which it opposes any change in current flowing through it.

Self Inductance (Coefficient of Self Induction)
The flux linked with a coil is directly propotional to the current flowing through it. $\phi \propto I$

$\phi = LI$

where L is a constant called Self Induc-

let $I = 1A$

Then $\phi = L$

∴ The Self inductance of a coil can be defined as the flux linked with the coil when the current flowing through it is 1 A

According to Faraday's law Induced EMF $E = -\frac{d\phi}{dt}$

$$= -\frac{d(LI)}{dt}$$

$$= -L \cdot \frac{dI}{dt}$$

let $\frac{dI}{dt} = 1\,A/s$

then $E = -L$

∴ Self Inductance of a coil is numerically equal to the EMF Induced in the coil when the rate of change of current through the coil is 1 A/s.

The unit of Self Inductance is Henry

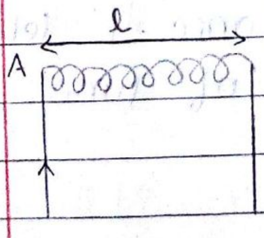
1 Henry = 1 Henry = $\frac{1\ \text{Volt}}{1\ \text{Ampere/sec}}$

∴ Self Inductance of a coil is said to be 1 Henry If an EMF of 1 V is Induced in it when the rate of change of current flowing through it is 1 A/s.

Expression for Self Inductance

Consider a solenoid of length l, area of cross section A and carrying current I. let N be the total no. of turns and n be the no. of turns per unit length.

Then $n = \frac{N}{l}$



The Magnetic field inside the solenoid is given by $B = \mu_0 n I$

we have the flux $\phi = N\,\vec{B}\cdot\vec{A}$

$\phi = NBA\cos\theta$.

$\phi = NBA \quad (\because \theta = 0)$

$\phi = N\mu_0 nIA$

$\frac{\phi}{I} = \mu N n A$

$\frac{\phi}{I} = \mu_0 n l \times n A$

$L = \mu_0 n^2 A l$

If the space inside the solenoid is filled with a medium of relative permeability μ_r, then $L = \mu_0 \mu_r n^2 A l$

But $n = N/l$

$L = \mu_0 \mu_r \frac{N^2}{l^2} . A . l$

$= \mu_0 \mu_r N^2 A / l$

The self inductance of a solenoid depends on the no. of turns of the solenoid, (ii) Area of cross section (iii) length of the solenoid (iv) Nature of material kept inside a solenoid

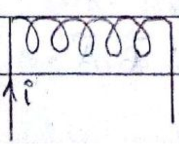
NOTE

→ The self inductance of a solenoid is $\propto$ to its length when the no. of turns per unit length remains constant

→ The self inductance of a solenoid is $\frac{1}{\propto}$ to its length when the total no. of turns remains constant.

Energy stored in an Inductor

Consider an inductor of self inductance 'L'. let i be the current at any instant of time.



Magnitude of Induced Emf of the coil is given by $E = L \frac{di}{dt}$

E is the work done per unit charge $\therefore$ the work done for a small charge dq is $dw = E\,dq$

$$= L \frac{di}{dt} \cdot dq$$

$$= L\,i\,di, \text{ where } i = \frac{dq}{dt}, \text{ the current.}$$

$\therefore$ The total work done $W = \int_0^I L\,i\,di$

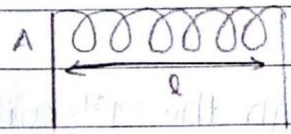
$$= L \int_0^I i\,di$$

$$= L \left[\frac{i^2}{2} \right]_0^I$$

$$W = \frac{1}{2} L I^2$$

This work done is stored as the energy of the inductor ie $U = \frac{1}{2} L I^2$

Energy density of an Inductor



Energy density is the energy per unit volume

$$\text{Energy density} = \frac{\text{Energy}}{\text{volume}}$$

$$= \frac{\frac{1}{2} L I^2}{A l}$$

$$= \frac{1}{2} \frac{\mu_0 n^2 A l I^2}{A l}$$

$$= \frac{1}{2} \mu_0 n^2 I^2 \times \frac{\mu_0}{\mu_0}$$

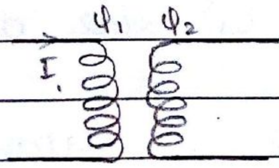
$$= \frac{1}{2} \frac{\mu_0^2 n^2 I^2}{\mu_0}$$

$$= \frac{1}{2} \frac{B^2}{\mu_0}.$$

Mutual Induction. whenever the magnetic flux linked with a coil changes due to the change in current flowing through it, an EMF is induced in the neighbouring coil. This phenomenon is called mutual induction.

when the current flowing through a coil increases. the flux linked with it also increases. As a result an EMF is induced in a neighbouring coil and it tries to decrease the growth of current in the 1st coil. similarly when the current in the first coil decreases an EMF is induced in the 2nd coil and it tries to oppose the decay of current in the first coil
∴ mutual induction can be defined as the property of a pair of coil by which each coil opposes any change in current flowing through the other.

Mutual Inductance
Coefficient of mutual inductance.



let I_1 be the current flowing through the 1st coil and ϕ_2 be the flux linked with the 2nd coil then $\phi_2 \propto I_1$.
$\phi_2 = M_{21} I_1$ —① where M_{21} is a constant called the ~~set~~ mutual inductance of 2nd coil with resp. to the 1st coil.

Similarly $\phi_1 \propto I_2$.
$\phi_1 = M_{12} I_2$ where M_{12} is the mutual inductance of 1st coil with resp. to the 2nd coil.

In general $\boxed{\phi = MI}$
where ϕ is the flux linked with one of the coil,

I is the current flowing through the other coil & M is the mutual inductance of the pair of the coil

let $I = 1A$,
then $\phi = M$

$\therefore$ mutual inductance of a pair of coil can be defined as the flux linked with one of the coil when ~~current~~ flux ~~flowin~~ current flowing through the other coil is 1A

we have induced EMF $E = -\frac{d\phi}{dt}$

$$= -\frac{d}{dt}(MI)$$

$$= -M\frac{dI}{dt}$$

let $\frac{dI}{dt} = 1A/s$ then $E = -M$

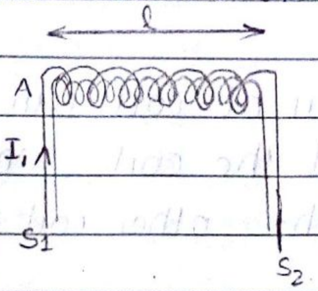
i.e the mutual inductance of a pair of coil is numerically equal to the EMF induced in one of the coil when the rate of change of current through the other coil is 1A/s.
The unit of mutual inductance is Henry.

$$1H = \frac{1V}{1A/s}.$$

The mutual inductance of a pair of coil is said to be 1 Henry, if an emf of 1v is induced in one of the coil when the rate of change of current through the other coil is 1 A/s.

Expression for Mutual inductance.

Consider two coaxial solenoids S_1 & S_2 with same length l and same area of cross section A. let N_1 & N_2 be the total no. of turns of S_1 and S_2 resp. and n_1 and n_2 be the no. of turns per unit length of S_1 and S_2 resp.



let I_1 be the current flowing through S_1. Then the magnetic field inside the solenoid

$B_1 = \mu_0 n_1 I_1$

$\therefore$ flux linked with second coil, $\phi_2 = N_2 \vec{B_1} \cdot \vec{A}$

$$\phi_2 = N_2 B_1 A \cos\theta$$

$$\phi_2 = N_2 B_1 A \quad (\because \theta = 0)$$

$$\phi_2 = N_2 \mu_0 n_1 I_1 A$$

$$\frac{\phi_2}{I_1} = \mu_0 N_2 n_1 A$$

$$M_{21} = \mu_0 n_2 l n_1 A$$

$$M_{21} = \mu_0 n_1 n_2 A l \text{ — ①}$$

let I_2 be the current flowing through S_2

$$B_2 = \mu_0 n_2 I_2$$

$$\phi_1 = N_1 \vec{B_2} \cdot \vec{A}$$

$$= N_1 B_2 A \cos\theta$$

$$= N_1 B_2 A$$

$$\phi_1 = N_1 \mu_0 n_2 I_2 A$$

$$\frac{\phi_1}{I_2} = \mu_0 N_1 n_2 A$$

$$M_{12} = \mu_0 n_1 n_2 A l \text{ — ②}$$

From ① and ② we get

$M_{12} = M_{21}$

$\therefore$ Mutual Inductance of a pair of coil

$M = \mu_0 n_1 n_2 A l$

if there is a medium in the coil $M = \mu_0 \mu_r n_1 n_2 A l$

But $n_1 = \frac{N_1}{l}$ & $n_2 = \frac{N_2}{l}$

$$M = \mu_0 \mu_r \frac{N_1}{l} \times \frac{N_2}{l} \times Al$$

$$\boxed{M = \mu_0 \mu_r N_1 N_2 A / l.}$$

NOTE : ~~Mutu~~. from the above results, It is clear that the mutual Inductance of a pair of coil is independent of the coil through which the current is passed.

Coupling

2 coils are said to be magnetically coupled if a part of flux is shared by the 2 coils.

let ϕ_1 be the flux linked with 1st coil and

ϕ_2 be the flux linked with 2nd coil then coefficient of coupling $k = \frac{\phi_2}{\phi_1}$

If $\phi_2 = \phi_1$ then $k = 1$, the coupling is called tight coupling

If $\phi_2 = 0$ then $k = 0$, ie there is no mutual Induction b/w coils

If $k < 1$ the coupling is called loose coupling

the part of the flux that links the two coils is called the mutual flux and the part of flux that does not links the two coils is called the leakage flux.

Relation b/w Mutual Inductance and self inductance

let M be the mutual Induction of a pair of coil having self Induction L_1 and L_2 then $\boxed{M = k\sqrt{L_1 L_2}.}$

If $L_1 = L_2 = L$ & $k = 1$

then $M = L$

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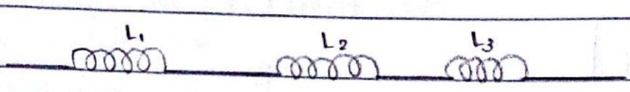
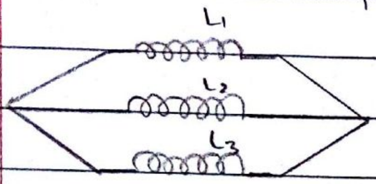
Combination of Inductors

(i) Inductors in series



$L = L_1 + L_2 + L_3$

(ii) Inductors in parallel

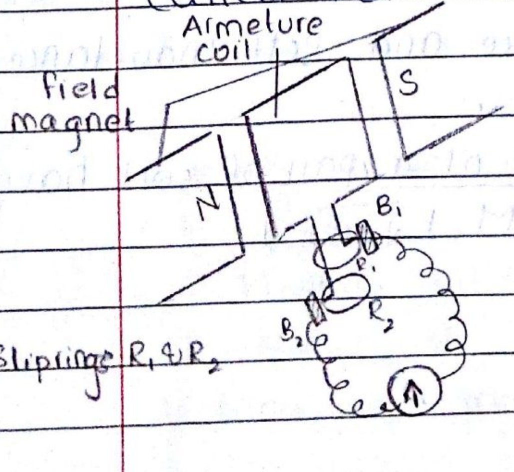


$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

AC Generator

A generator is a device used to convert mechanical energy to electrical energy
It works on the principle of Electromagnetic Induction.

WORKING.
A generator consists of an Armature coil, rotating in between the pole pieces of a powerful magnet called field magnets.

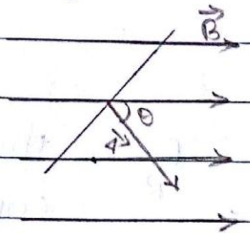


when the coil is rotating in magnetic fields an EMF is induced in it due to the change in flux linked with it. The induced current can be taken out to the external circuit through the arrangement of Sliprings and brushes

Expression for Induced EMF and Induced current In an AC generator

Consider an Armeture coil of area A and no. of turns 'N'. rotating In a uniform magnetic field of Intensity B with a uniform angular velocity ω. let the coil rotales through an angle θ in a time t, then $\omega = \frac{\theta}{t}$

$\theta = \omega t$



we have the flux linked the coil,

$\phi = N \vec{B}.\vec{A}$ where $\vec{A}$ is the area vector

$\phi = NBA\cos\theta$.

$\phi = NBA\cos\omega t$

we have the Induced EMF $E = -\frac{d\phi}{dt}$

$\therefore E = -\frac{d}{dt}(NBA\cos\omega t)$

$= -NBA \frac{d}{dt}\cos\omega t$

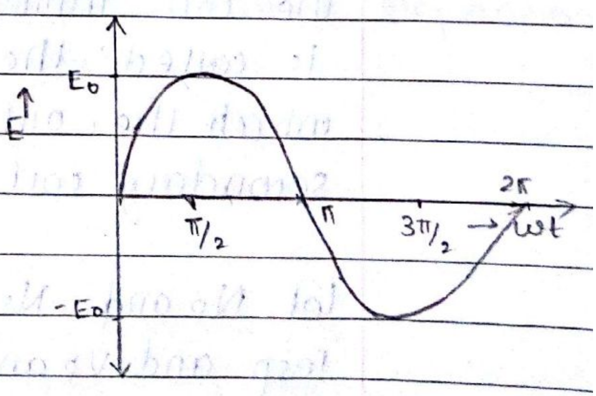
$= -NBA \times -\sin\omega t \times \omega$

$E = NBA\omega \sin\omega t$.

$\boxed{E = E_0 \sin\omega t}$ where $E_0 = NBA\omega$, peak Value of EMF.

The variation of Induced EMF with ωt is graphically shown In the figure.

ωt	0	$\pi/2$	π	$3\pi/2$	2π
E	0	E_0	0	$-E_0$	0



This complete set of variation for 1 rotation of coil is called one cycle of AC.

No. of cycles generated per second is called the frequency of Ac and the time taken to complete 1 cycle of Ac is called the timeperiod of Ac.

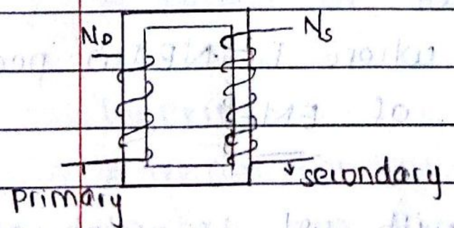
let R be the resistance of the coil, then Induced current $I = \frac{E}{R}$

$= \frac{E_0 \sin \omega t}{R}$

$I = I_0 \sin \omega t$, where $I_0 = \frac{E_0}{R}$, the peak value of current.

TRANSFORMER

Transformer is a device used to increase or decrease Ac voltage. It works on the principle of Mutual induction.
It consists of two coils wound on a rectangular iron core as shown in the figure



The coil through which the input voltage is given is called the primary coil and the coil through which the output voltage is taken is called the secondary coil.

let N_p and N_s be the no. of turns in 1° and 2° resp. and V_p and V_s the ~~voltage~~ primary and secondary voltages resp. let ϕ be the flux linked with both the coils. Then $V_p = N_p \cdot \frac{d\phi}{dt}$ —— ①

$$V_s = N_s \cdot \frac{d\phi}{dt} \text{ —— ②}$$

② ÷ ①

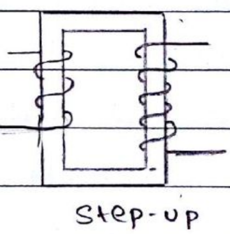
$$\frac{V_s}{V_p} = \frac{N_s \frac{d\phi}{dt}}{N_p \cdot \frac{d\phi}{dt}}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

This result is known as the transformer ratio.

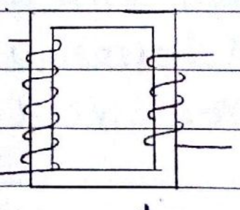
Case-1

If $N_s > N_p$ then $V_s > V_p$. i.e the output voltage is greater than input voltage. Such a transformer is called step-up transformer


step-up

Case-2

If $N_s < N_p$, $V_s < V_p$. i.e output voltage is less than input voltage. Such a transformer is called step-down transformer


step-down

efficiency of a transformer

efficiency of a transformer can be defined as the ratio of output power to the input power. i.e

$$\eta = \frac{P_{out}}{P_{in}}$$

$$= \frac{V_s I_s}{V_p I_p}$$ where I_p and I_s are the 1° and 2° currents resp.

If $P_{out} = P_{in}$

$$\eta = 1 \quad x/x$$

$$= 100\%$$

i.e there is no power loss and such a transformer is known as an Ideal transformer.

In an Ideal transformer

$$V_s I_s = V_p I_p$$

i.e $$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

$$\therefore \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

Transformer losses

1. Copper loss
2. Iron loss (Eddy Current loss)
3. Hysterisis loss
4. Flux leakage loss
5. Humming loss.